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NONLINEAR SUBGRID-SCALE MODELS FOR LARGE-EDDY SIMULATION OF ROTATING TURBULENT FLOWS

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INTRODUCTION

Rotating turbulent flows form a challenging test case for commonly used eddy viscosity models. We therefore consider subgrid-scale models with an additional term, which is nonlinear in the local velocity gradient. We show that this nonlinear model term leads to improved predictions of energy transfer in rotating homogeneous isotropic turbulence. We also investigate the role of nonlinear subgrid-scale models in rotating channel flows.

NONLINEAR SUBGRID-SCALE MODELS

In large-eddy simulation one seeks to predict the behavior of the larger scales of motion within a flow field. It is common to distinguish between large and small scales of motion using a filtering operation, denoted by an overbar in what follows. The evolution of rotating large-scale velocity fields can be described by the filtered Navier-Stokes equations in a rotating frame,

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + 2\epsilon_{ijk}\bar{u}_j\Omega_k - \frac{\partial}{\partial x_j}\tau_{ij}. \tag{1}$$

In this work, we focus on incompressible flows. The turbulent, or subgrid-scale, stresses, $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j$, represent the interactions between large and small scales of motion. As they are not solely expressed in terms of the filtered velocity field, they have to be modeled.

We consider subgrid-scale models of the nonlinear form,

$$\tau^{\mathrm{mod}} - \frac{1}{3}\operatorname{tr}(\tau^{\mathrm{mod}})I = -2\nu_e S + \mu_e(S\Omega - \Omega S). \tag{2}$$

Here, S and Ω represent the rate-of-strain and rate-of-rotation tensors, respectively. The first term on the right-hand side of Eq. (2), the usual eddy viscosity term, parametrizes dissipative processes in turbulent flows. The second term, that is nonlinear in the velocity gradient, is added because it is perpendicular to the rate-of-strain tensor. Therefore, it does not directly contribute to the subgrid dissipation and it represents energy transport. As this term contains the rate-of-rotation tensor, it has “a particular potential for [the simulation of] rotating flows” [1].

The eddy viscosity, ν_e, and transport coefficient, μ_e, are taken to depend on the vortex stretching magnitude, a quantity that can be used to correct the near-wall scaling and dissipation behavior of the Smagorinsky model [2, 3].

NUMERICAL RESULTS

The nonlinear subgrid-scale model of Eq. (2), with vortex-stretching-based eddy viscosity and transport coefficient [2, 3], was used to perform large-eddy simulations of rotating and nonrotating decaying homogeneous isotropic turbulence, and of spanwise-rotating turbulent plane-channel flow. These simulations were performed using an incompressible Navier-Stokes solver that employs a kinetic-energy-conserving spatial discretization of finite-volume type [4]. Here, also the Coriolis force was implemented in an energy-conserving fashion.

Rotating homogeneous isotropic turbulence

The simulations of rotating decaying homogeneous isotropic turbulence were set up according to the experiment by Comte-Bellot and Corrsin [5], with the addition of the Coriolis force [6]. For the large-eddy simulations we employed a uniform, isotropic 64^3 grid. Apart from the nonlinear subgrid-scale model of Eq. (2), we consider results from direct numerical simulations on a 512^3 grid, and from large-eddy simulations without a model and with the dynamic Smagorinsky model [6].

Figure 1 shows the resulting three-dimensional kinetic energy spectra for rotation numbers $Ro = 0$ (no rotation) and $Ro = 100$ (significant rotation). As expected, the Coriolis force makes the transport of energy to larger scales of motion grow. Due to this, the rate of dissipation of kinetic energy decreases with increasing rotation rate. Neither the eddy viscosity model that is obtained when only considering the first term on the right-hand side of Eq. (2) (again with a vortex-stretching-based eddy viscosity), nor the dynamic Smagorinsky model capture properly the large-scale energy content of both the rotating and the nonrotating flow. The nonlinear model term can, however, correct this behavior, showing its ability to transfer energy. To that end, we observe that the nonlinear model term turns off with increasing rotation rate. Therefore, the eddy viscosity was tuned to provide a proper prediction of the dissipation for a rotation number of $Ro = 100$. The nonlinear model term, was subsequently used to correct the large-scale energy distribution in the nonrotating ($Ro = 0$) case.

Spanwise-rotating plane-channel flow

We also performed large-eddy simulations of spanwise-rotating plane-channel flow at friction Reynolds number

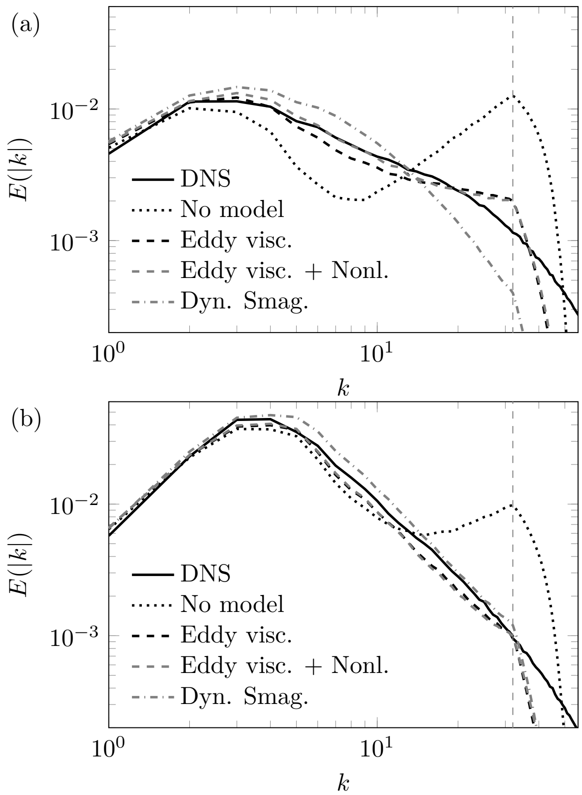

Figure 1: Three-dimensional kinetic energy spectra as a function of computational wavenumber for homogeneous isotropic turbulence at rotation number (a) $Ro = 0$ and (b) $Ro = 100$.

$Re_\tau = 180$ and rotation number $Ro^+ = 22$. These simulation were set up according to the work of Grundestam et al. [7]. In this case, large-eddy simulations were performed on a $64 \times 32 \times 64$ grid that was stretched in the wall-normal direction. Reference data come from direct numerical simulations on a $256 \times 192 \times 192$ grid, as well as from large-eddy simulations without a model and with the dynamic Smagorinsky model [6].

Figure 2 shows simulation results for the mean velocity profile and the streamwise deviatoric Reynolds stresses compensated for by the average model contribution. Although Re_τ is relatively small (i.e., 180), results obtained from direct numerical simulations and large-eddy simulations without a model are quite distinct. This underlines the need for subgrid-scale modeling at the current resolution. Large-eddy simulations using the currently considered eddy viscosity models, however, only qualitatively agree with the results from the direct numerical simulation, while quantitative improvements are necessary. A nonlinear model term may again be of help, although the vortex-stretching-based nonlinear term considered here only allows for marginal improvements over results obtained using eddy viscosity models alone (spread indicated in Figure 2). We therefore plan to perform additional simulations of rotating channel flows, in which we aim to test different nonlinear subgrid-scale models, all obtained from a class of subgrid-scale models that are consistent with important properties of the Navier-Stokes equations and the turbulent stresses [8].

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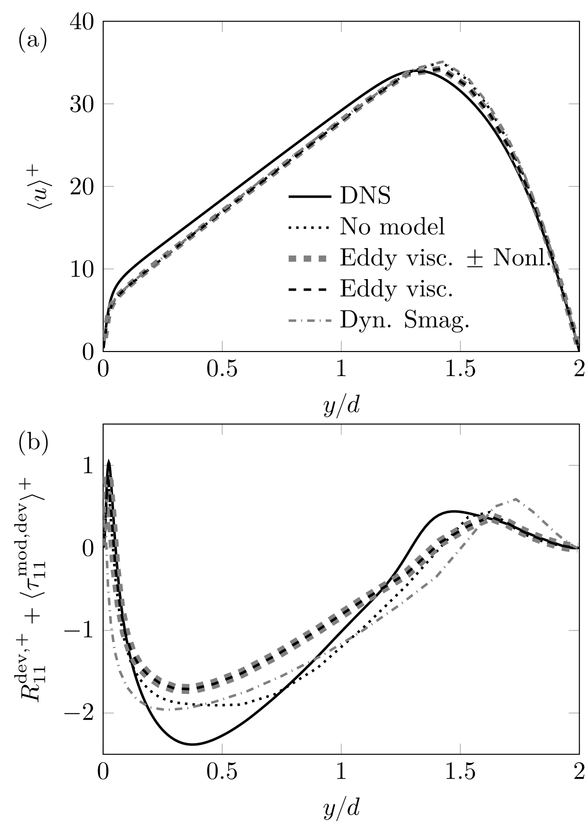

Figure 2: Mean velocity profile (a) and deviatoric streamwise Reynolds stresses compensated by the model contribution (b) for plane-channel flow at friction Reynolds number $Re_\tau = 180$ and rotation number $Ro^+ = 22$.