Going beyond eddy viscosity: Finding a minimal representation of subgrid-scale stresses in large-eddy simulation

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Going beyond eddy viscosity: Finding a minimal representation of subgrid-scale stresses in large-eddy simulation

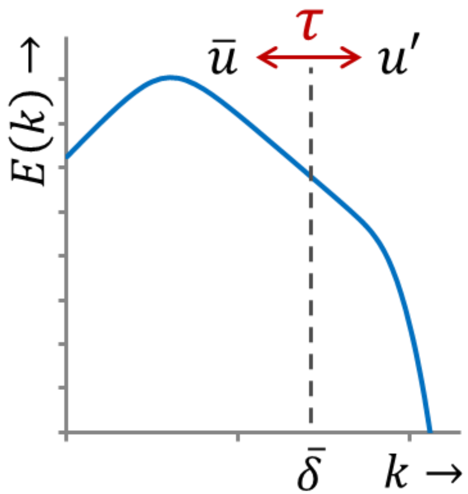
Construct a general nonlinear subgrid-scale model

Discuss model requirements from exact properties of the Navier-Stokes equations

Introduction

Research topic

- Large-eddy simulation of incompressible turbulent flows



- Creating subgrid-scale models for large-eddy simulation

Pope (2011), *Turbulent Flows*
Sagaut (2005), *Large Eddy Simulation for Turbulent Flows*
Berselli, Iliescu and Layton (2006), *Mathematics of Large Eddy Simulation of Turbulent Flows*

Introduction

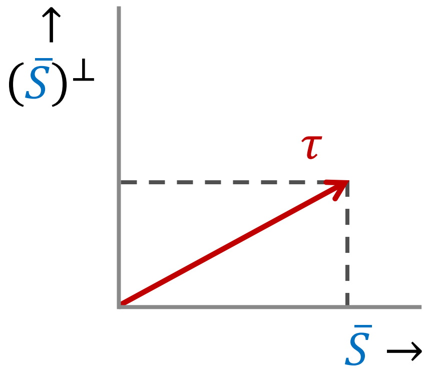
Example: eddy viscosity model

- Linear constitutive relation

$$\tau^{mod} - \frac{1}{3}\mathrm{tr}\left(\tau^{mod}\right) I = -2\nu_e \bar{S}$$

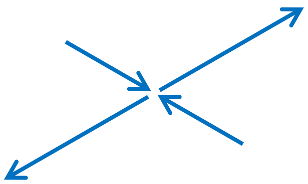
  – Models turbulence as a dissipative process

- Problem[1]



$\bar{S}$

rate of strain



deformations

Capture nondissipative processes

1. Tao, Katz and Meneveau (2002), *J. Fluid Mech.* **457**, 35

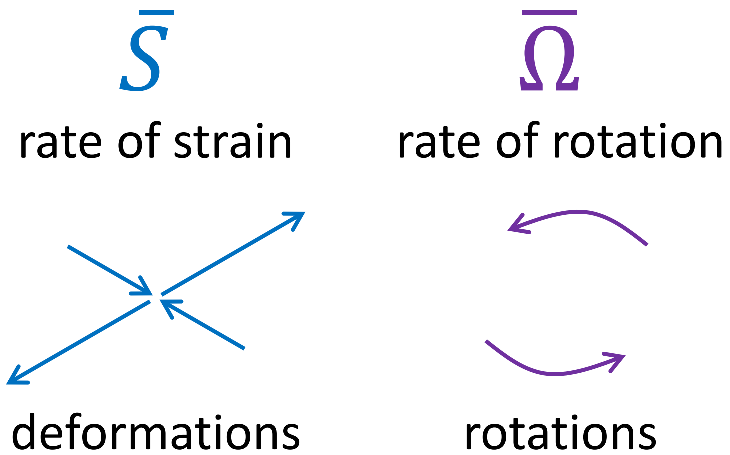
Constructing a general nonlinear subgrid-scale model

Starting point

- General constitutive relation

$$\tau^{mod} = f(\bar{S}, \bar{\Omega})$$

- Building blocks



LES: Lund and Novikov (1992), *CTR Ann. Res. Briefs*, 27
RANS: Pope (1975), *J. Fluid Mech.* **72**, 331,
Gatski and Jongen (2000), *Progr. Aerosp. Sci.* **36**, 655

Constructing a general nonlinear subgrid-scale model

All possible terms

I $\quad \bar{S}$ $\quad \bar{S}^2$ $\quad \bar{S}^3$ $\quad \cdots$

$\bar{\Omega}$ $\quad \bar{S}\,\bar{\Omega}$ $\quad \bar{S}^2\,\bar{\Omega}$

$\bar{\Omega}\,\bar{S}$

$\bar{\Omega}^2$ $\quad \bar{\Omega}^2\,\bar{S}$ $\quad \ddots$

$\vdots$

Number of terms
∞

LES: Lund and Novikov (1992), *CTR Ann. Res. Briefs*, 27
RANS: Pope (1975), *J. Fluid Mech.* **72**, 331,
Gatski and Jongen (2000), *Progr. Aerosp. Sci.* **36**, 655

Constructing a general nonlinear subgrid-scale model

Symmetric terms + Cayley-Hamilton theorem

I $\quad \bar{S}$ $\quad \bar{S}^2$

$\bar{S}\,\bar{\Omega} - \bar{\Omega}\,\bar{S}$

$\bar{\Omega}^2$ $\quad \bar{S}^2\,\bar{\Omega} - \bar{\Omega}\,\bar{S}^2$

$\ddots$

Number of terms
11

A matrix satisfies its own characteristic equation

LES: Lund and Novikov (1992), *CTR Ann. Res. Briefs*, 27
RANS: Pope (1975), *J. Fluid Mech.* **72**, 331,
Gatski and Jongen (2000), *Progr. Aerosp. Sci.* **36**, 655

Model tests: Wang and Bergstrom (2005), *Phys. Fluids* **17**, 035109
Marstorp *et al.* (2009), *J. Fluid Mech*. **639**, 403

Constructing a general nonlinear subgrid-scale model

Subgrid-scale stresses

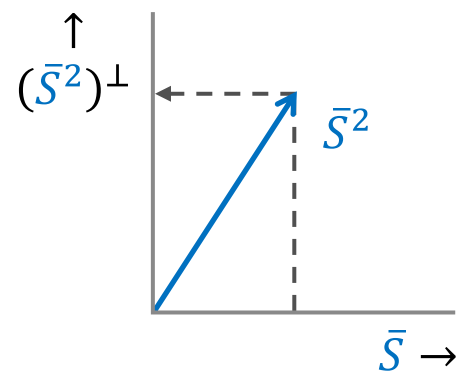
$$\tau = \begin{bmatrix} * & * & * \\ & * & * \\ & & * \end{bmatrix}$$

Degrees of freedom
6

- Interested in nondissipative processes

Subtract $\bar{S}$ from other terms

Make all other terms independent



Constructing a general nonlinear subgrid-scale model

Resulting independent model terms

I $\quad \bar{S} \quad (\bar{S}^2)^{\perp}$

$\bar{S}\,\bar{\Omega} - \bar{\Omega}\,\bar{S}$

$(\bar{\Omega}^2)^{\perp} \quad \ddots$

At most 6 nonzero terms

- General nonlinear constitutive relation

$$\tau^{mod} = \alpha_0 I + \alpha_1 \bar{S} + \alpha_2 (\bar{S}^2)^{\perp} + \alpha_3 (\bar{\Omega}^2)^{\perp} + \cdots$$

  – Coefficients?

General representation for any symmetric 3 x 3 matrix

Model requirements

Exact properties of the Navier-Stokes equations

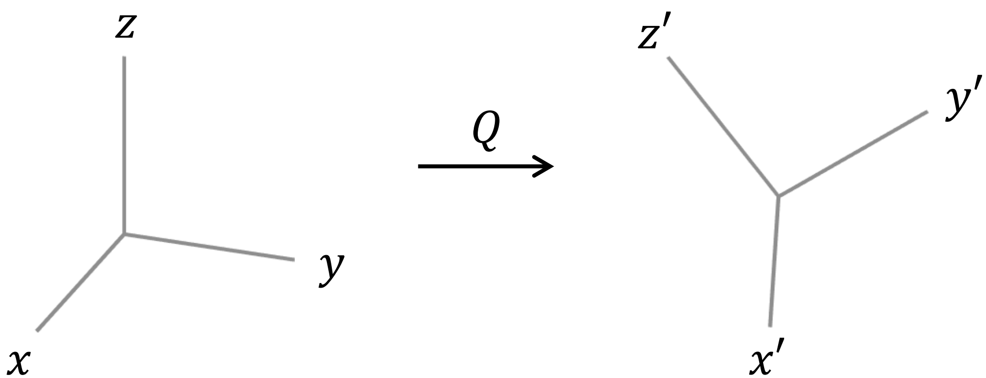
- Invariances of the Navier-Stokes equations[1,2]
  - Rotations and reflections
  - Time and pressure translation, generalized Galilean invariance
  - Scaling transformations
  - Two-dimensional material frame-indifference (2D MFI)
  - Time-reversal invariance (inviscid flow)
- Vreman's subgrid dissipation requirements[3]
- Near-wall scaling behavior[4]
- …

Tune model coefficients to satisfy these properties

1. Oberlack (1997), *CTR Ann. Res. Briefs*, 3
2. Razafindralandy, Hamdouni and Oberlack (2007), *Eur. J. Mech. B-Fluid.* **26**, 531
3. Vreman (2004), *Phys. Fluids* **16**, 3670
4. Chapman and Kuhn (1986), *J. Fluid Mech.* **170**, 265

Model requirements

Rotational invariance



$\bar{S} \to Q\bar{S}Q^T$ ✓

$\bar{\Omega} \to Q\bar{\Omega}Q^T$ ✓

- Requirement on model coefficients: $\alpha_i \to \alpha_i$

> 💡 Make coefficients a function of local flow properties

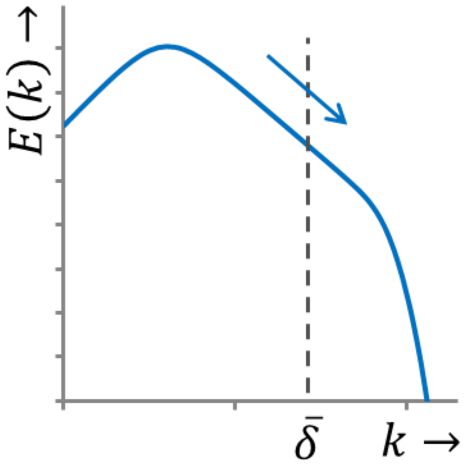
$\mathrm{tr}(\bar{S}^2), \mathrm{tr}(\bar{\Omega}^2), \mathrm{tr}(\bar{S}\,\bar{\Omega}^2), \cdots$

- Further restrictions follow from other symmetries

Model requirements

Vreman's subgrid dissipation requirements

- Laminar flow: *no* creation of small scales by turbulence model
- Turbulent flow: creation of small scales …
- Importance: no unphysical transition due to model

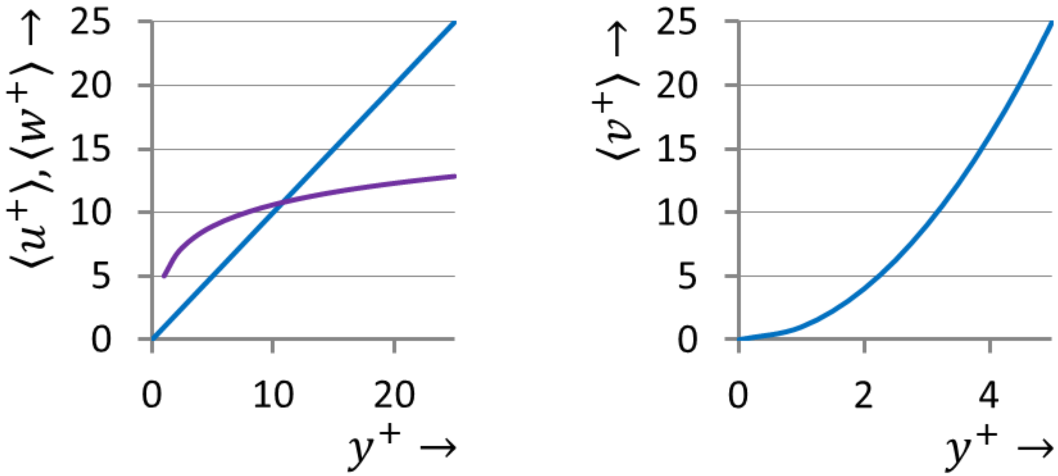


- Restricts $\alpha_1(\mathrm{tr}(\bar{S}^2), \mathrm{tr}(\bar{\Omega}^2), \cdots)\ \bar{S}$

Vreman (2004), *Phys. Fluids* **16**, 3670

Model requirements

Near-wall scaling behavior



$$\langle \tau_{ij} \rangle \propto (y^+)^{\cdot\cdot}$$

- Model should capture this behavior
  - Not too much dissipation in near-wall region
- Restriction of coefficients
  - $\alpha_1 \propto (y^+)^3, \cdots$

Chapman and Kuhn (1986), *J. Fluid Mech.* **170**, 265

Model requirements

Analysis of existing models

Requirement	Smago-rinsky[1]	WALE[2]	Vreman[3]	QR[4]	S3PQR[5]
Rotations, reflections	✓	✓	✓	✓	✓
Translations, Galilean	✓	✓	✓	✓	✓
Scaling	✓*	✓*	✓*	✓*	✓*
2D MFI	✓	✗	✗	✓	✓**
Laminar flow $\nu_e = 0$	✗	✗	✗	✓	✓**
Turbulent flow $\nu_e \neq 0$	✓	✓	✓	✗	✗
Near-wall behavior	$\mathcal{O}(y^0)$	$\mathcal{O}(y^3)$	$\mathcal{O}(y^1)$	$\mathcal{O}(y^1)$	$\mathcal{O}(y^3)$
Time reversal	✓*	✓*	✓*	✓*	✓*/**

* If the dynamic procedure is used
** Additional restrictions apply

1. Smagorinsky (1963), *Mon. Weather Rev.* **91**, 99
2. Nicoud and Ducros (1999), *Flow, Turbul. Combust.* **62**, 183
3. Vreman (2004), *Phys. Fluids* **16**, 3670
4. Verstappen, Rozema and Bae (2014), *CTR Proc. Summ. Progr.*, 417
5. Trias *et al.* (2015), *Phys. Fluids* **27**, 065103

Model requirements

Example model

- Linear constitutive relation

$$\tau^{mod} - \tfrac{1}{3}\mathrm{tr}\left(\tau^{mod}\right) I = -2\nu_e \bar{S}$$

$$\nu_e = c\bar{\delta}^2 \frac{\mathrm{tr}\left(\bar{S}^3\right)^3}{\mathrm{tr}\left(\bar{S}^2\right)^4}$$

Requirement	
Rotations, reflections	✓
Translations, Galilean	✓
Scaling	✓*
2D MFI	✓
Laminar flow $\nu_e = 0$	✓
Turbulent flow $\nu_e \neq 0$	✗
Near-wall behavior	$\mathcal{O}(y^3)$
Time reversal	✓

* If the dynamic procedure is used

Conclusions & Discussion

Key points

- Constructed a general nonlinear subgrid-scale model
- Discussed model requirements from Navier-Stokes equations

Class of nonlinear subgrid-scale models

Future ideas

- Consistency with the second law of thermodynamics[1]..?
- Require realizability of the turbulent stresses[2]..?
- …

More exact properties of the Navier-Stokes equations?

More physical requirements for turbulence models?

1. Razafindralandy, Hamdouni and Oberlack (2007), *Eur. J. Mech. B-Fluid.* **26**, 531
2. Vreman, Geurts and Kuerten (1994), *J. Fluid Mech.* **278**, 351

Thank you for your attention!

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