# A new turbulence model for large-eddy simulations of rotating flows

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# Introduction

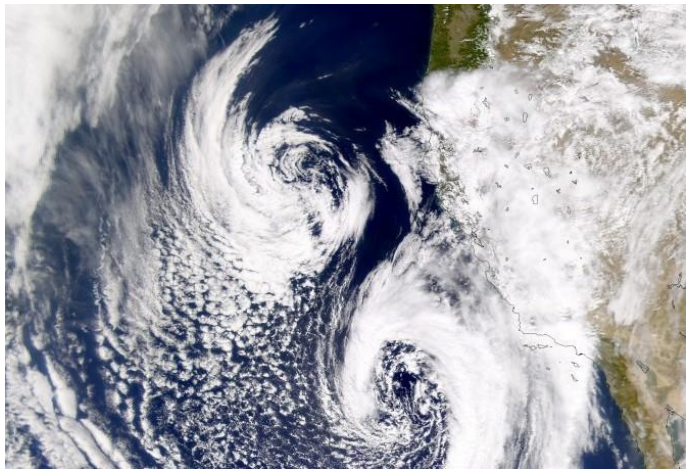
## Conclusions

- Rotating turbulent flows form a rich and challenging test case for large-eddy simulation

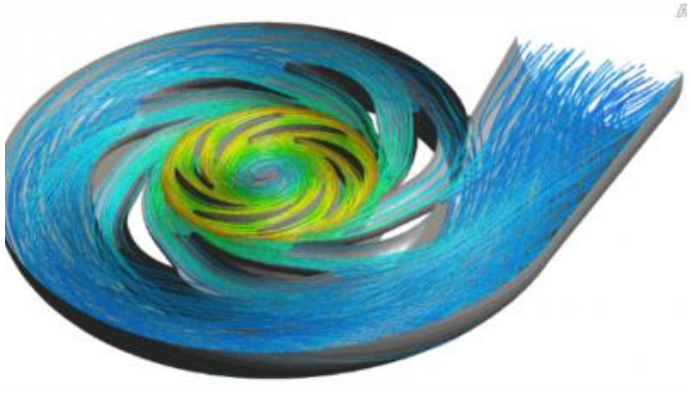
- Our new nonlinear subgrid-scale model improves predictions of the Reynolds stress anisotropy

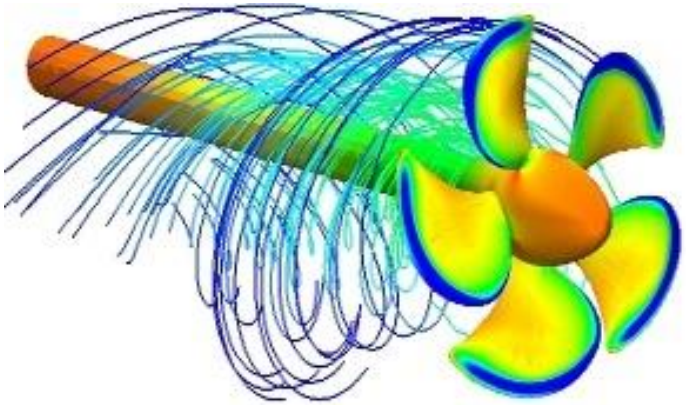
# Introduction

## Rotating turbulent flows



Ubiquitous in nature and engineering

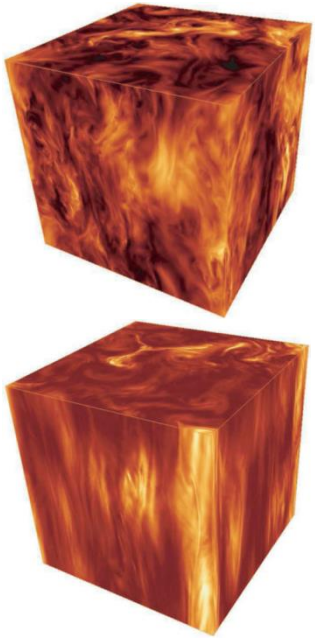




# Introduction

## Challenge

- Coriolis force
  - Conserves total kinetic energy
  - Transports energy from small to large scales of motion, reduces viscous dissipation of kinetic energy
  - Anisotropy



## Why a challenge?

- Large-eddy simulation
  - Eddy viscosity models: focus on dissipation
  - Our focus: also capture transport effects

Thiele & Müller (2009), *J. Fluid Mech.* **637**, 425

# Large-eddy simulation

## Large-eddy simulation of rotating turbulent flows

- Navier-Stokes equations in a rotating frame

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_i u_j\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + 2\nu\frac{\partial}{\partial x_j}S_{ij} + 2\epsilon_{ijk}u_j\Omega_k$$

convection pressure diffusion Coriolis

- Large-eddy simulation without explicit filtering
  - Coarse grid
  - Extra forcing term

$$-\frac{\partial}{\partial x_j}\tau_{ij}^{\mathrm{mod}}$$

model

# Large-eddy simulation

## A new nonlinear subgrid-scale model

$$\tau^{\mathrm{mod}} = \underbrace{-2\nu_{\mathrm{e}} S}_{\text{eddy viscosity}} + \underbrace{\mu_{\mathrm{e}}(S\,\Omega - \Omega\,S)}_{\text{nonlinear}}$$

- Describes dissipation
- Models well established
- 'Stable part' of gradient model
- Nondissipative

- Nondynamic coefficients based on vortex stretching magnitude

$$\nu_{\mathrm{e}} = C_\nu^2 \delta^2 \sqrt{2|S|} * f(|S\vec{\omega}|^2) \qquad \mu_{\mathrm{e}} = C_\mu \delta^2 * g(|S\vec{\omega}|^2)$$

- Nonlinear term represents energy transport in rotating homogeneous isotropic turbulence

Silvis *et al.* (2017), *Phys. Fluids* **29**, 015105
Silvis *et al.* (2016), *APS DFD Meeting*

# Large-eddy simulation

## Code

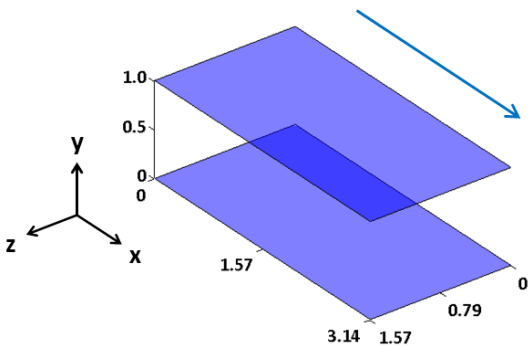
- Incompressible Navier-Stokes solver
- Finite-volume method on a staggered grid
- Pressure projection method
- One-leg explicit second-order time integration method

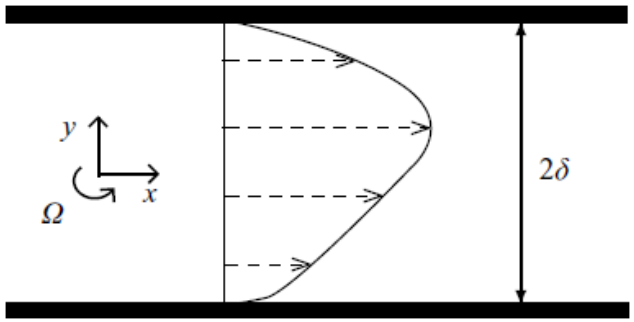
- Second-order symmetry-preserving spatial discretization
  - Kinetic energy conservation by …
    - convection
    - Coriolis force
    - nonlinear term of subgrid-scale model

Verstappen & Veldman (2003), *J. Comput. Phys.* **187**, 343
Beddhu *et al.* (1996), *J. Comput. Phys.* **128**, 427
Remmerswaal (2016), MSc thesis, University of Groningen

# Spanwise-rotating plane-channel flow

## Description




- Domain size: $2\pi d \times 2d \times \pi d$
- Periodic in $x$ and $z$ directions
- Constant pressure gradient in $x$ direction
- Rotation about the spanwise axis

- Characterized by

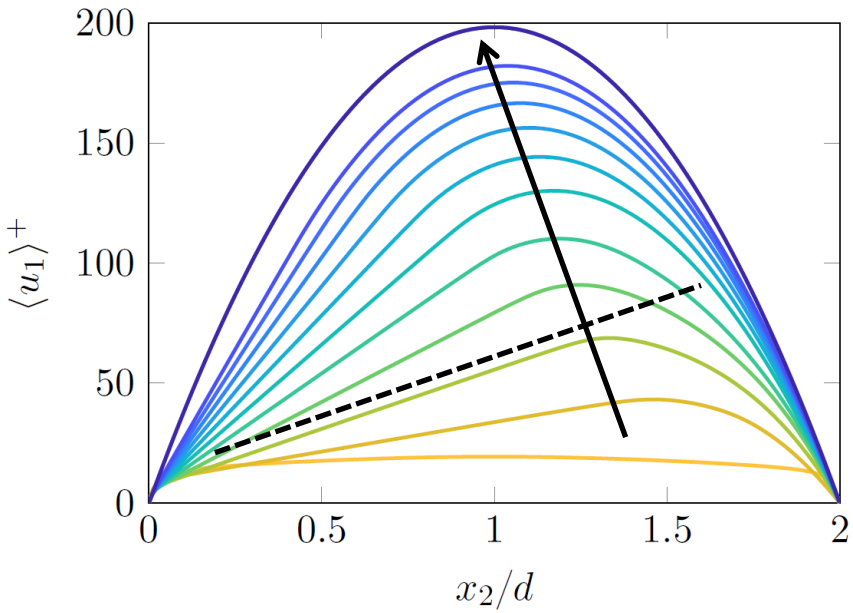
$$Re_\tau = \frac{u_\tau d}{\nu} \approx 395 \qquad Ro^+ = \frac{2\Omega d}{u_\tau} = 0 - 1000$$

Grundestam *et al.* (2008), *J. Fluid Mech.* **598**, 177
Brethouwer (2017), *J. Fluid Mech.* **828**, 424

# Spanwise-rotating plane-channel flow

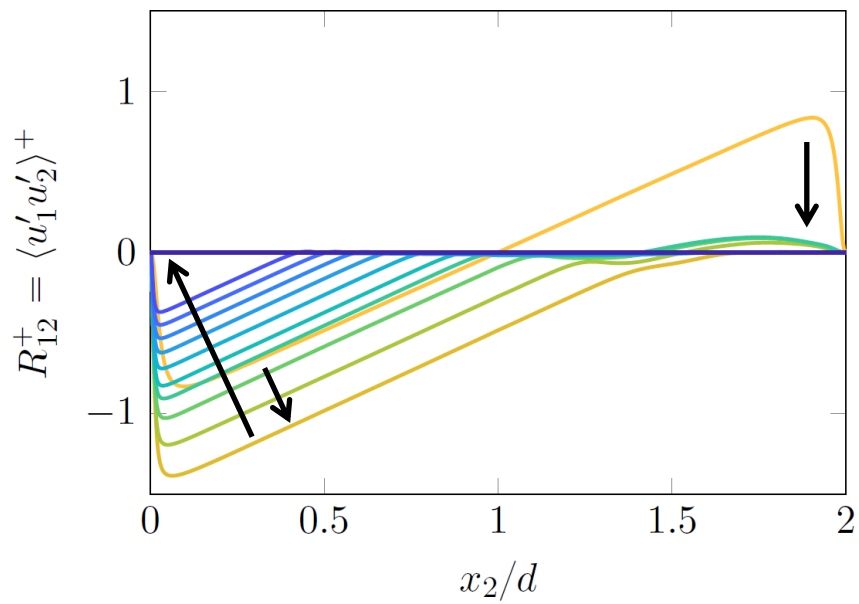
## Typical observations: $Ro^+$ dependence

- Mean velocity




  - Linear slope $\sim Ro^+$
  - Flow laminarizes with $Ro^+$

- Reynolds shear stress




  - 'Turbulent' and 'laminar' side
  - Flow laminarizes with $Ro^+$

$Re_\tau \approx 395$, $Ro^+ = 0 - 1000$
$n_x \times n_y \times n_z = 128 \times 256 \times 128$

# Spanwise-rotating plane-channel flow

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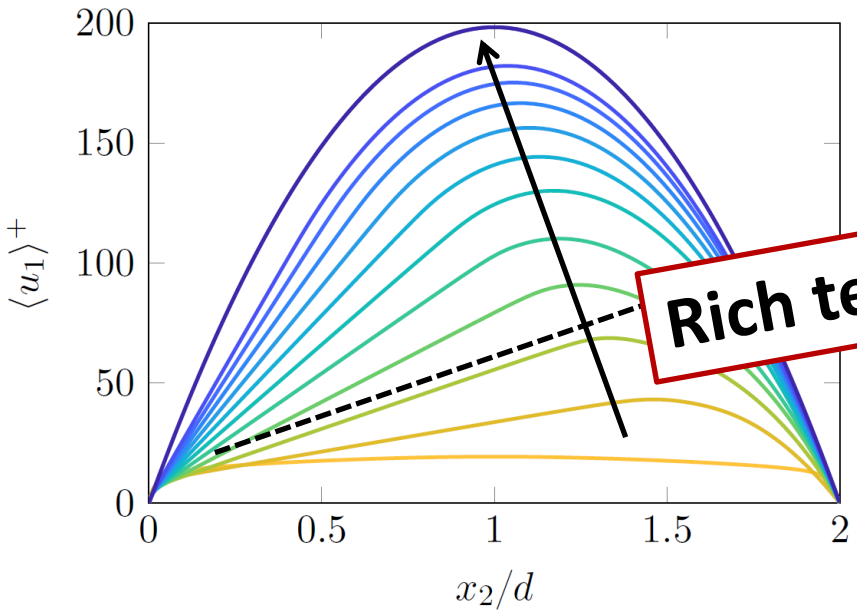


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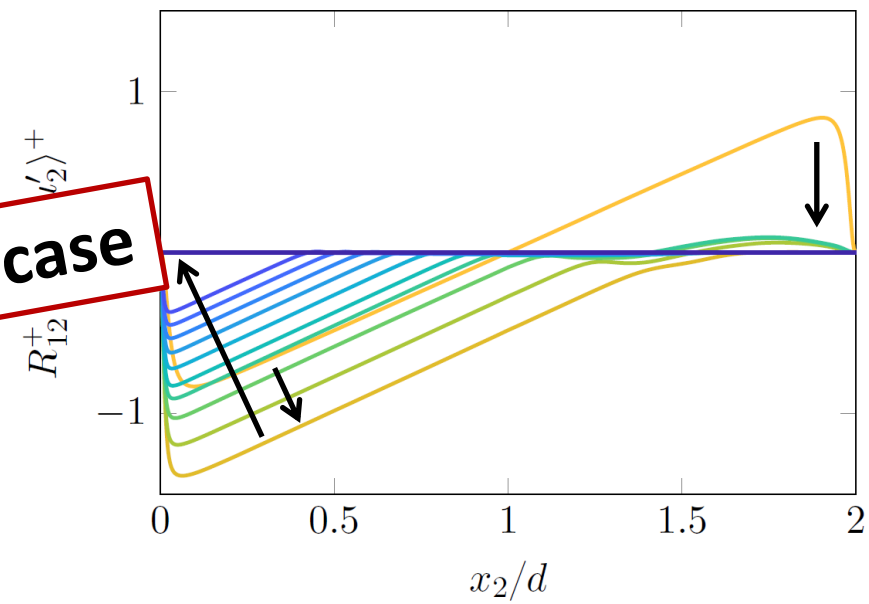


**Rich test case**

- Linear slope $\sim Ro^+$
- Flow laminarizes with $Ro^+$

- 'Turbulent' and 'laminar' side
- Flow laminarizes with $Ro^+$

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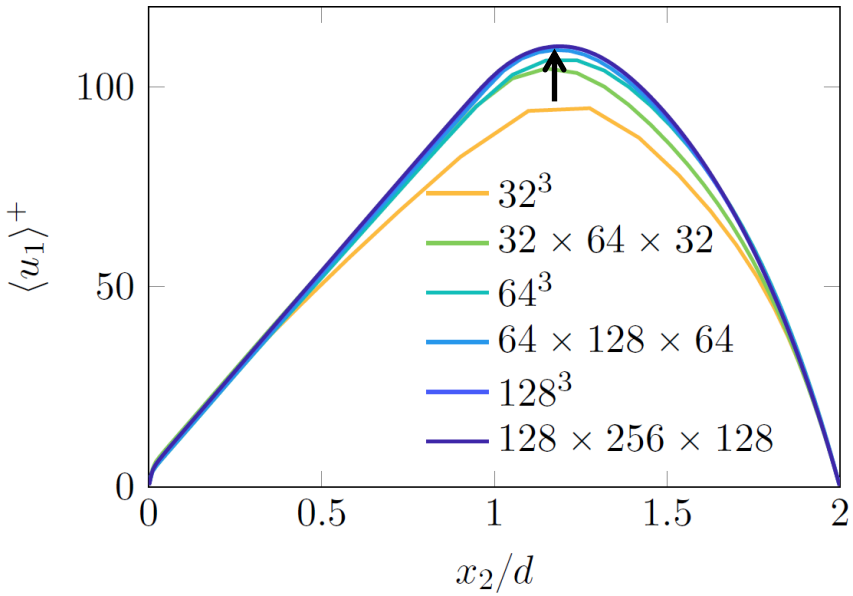
# Spanwise-rotating plane-channel flow

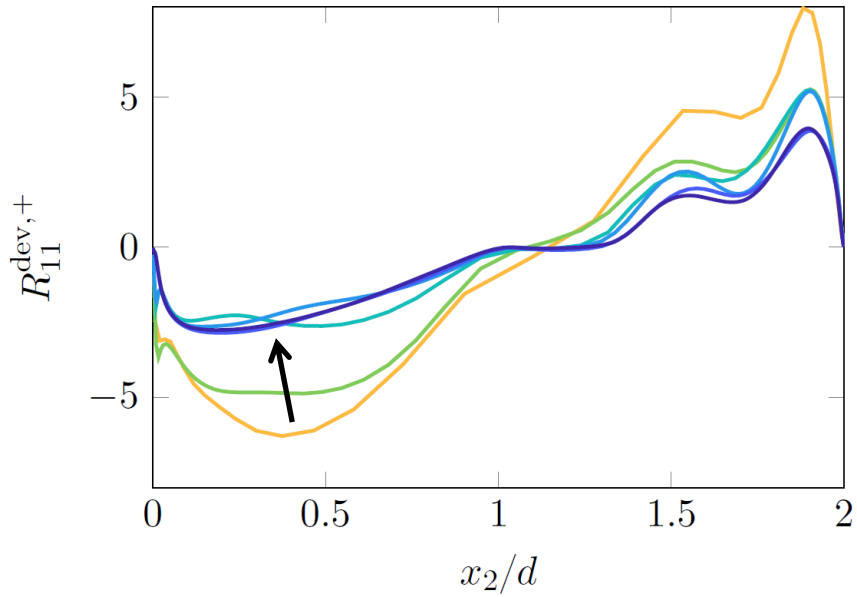
## Typical observations: resolution dependence

- Mean velocity




  - Ample opportunity for LES, even at $Re_\tau \approx 395$

- Streamwise stress anisotropy




  - Traceless subgrid-scale models: Reynolds stress anisotropy
  - Turbulent bursts
  - Focus on 'turbulent' side

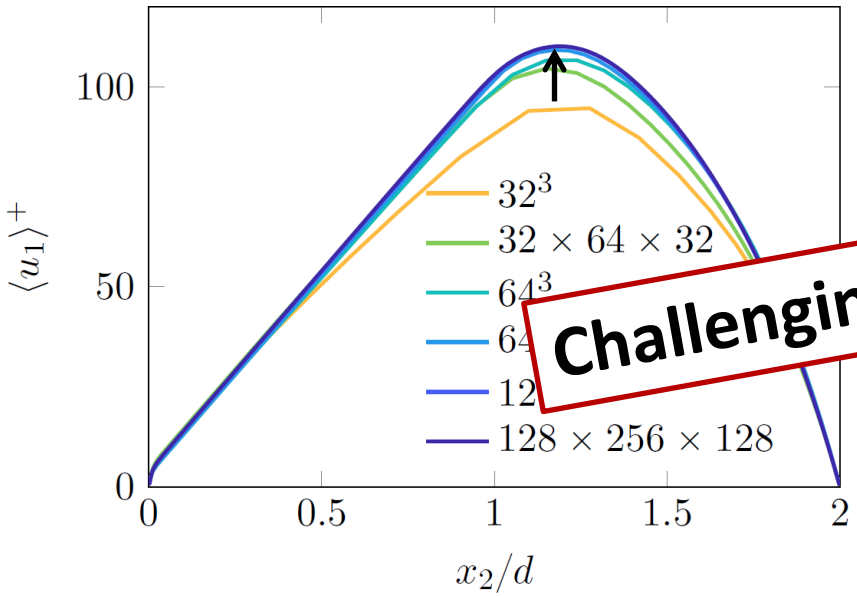
$Re_\tau \approx 395$, $Ro^+ = 100$

Winckelmans *et al.* (2002), *Phys. Fluids* **14**, 1809
Brethouwer (2017), *J. Fluid Mech.* **828**, 424

# Spanwise-rotating plane-channel flow

## Typical observations: resolution dependence

- Mean velocity

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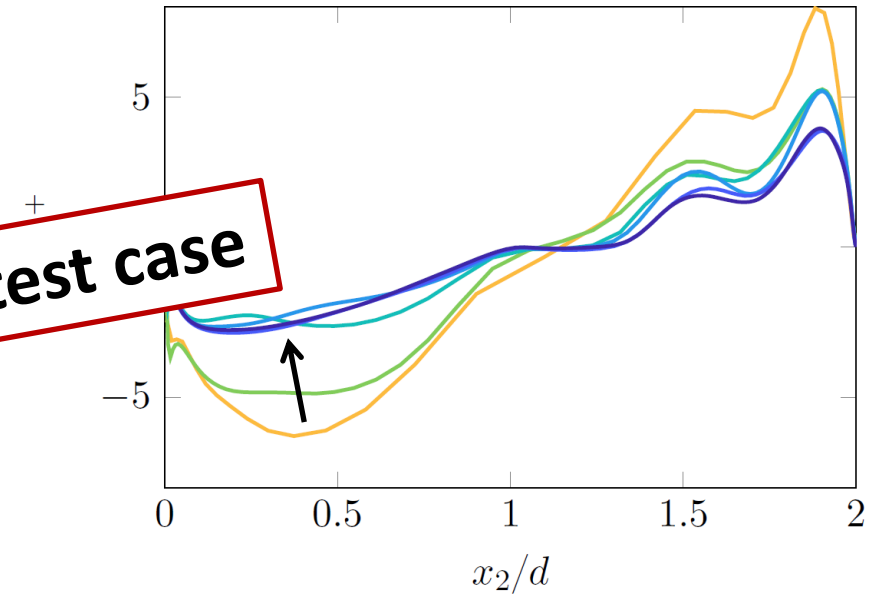


**Challenging test case**

- Traceless subgrid-scale models: Reynolds stress anisotropy
- Turbulent bursts
- Focus on 'turbulent' side

$Re_\tau \approx 395$, $Ro^+ = 100$

Winckelmans *et al.* (2002), *Phys. Fluids* **14**, 1809
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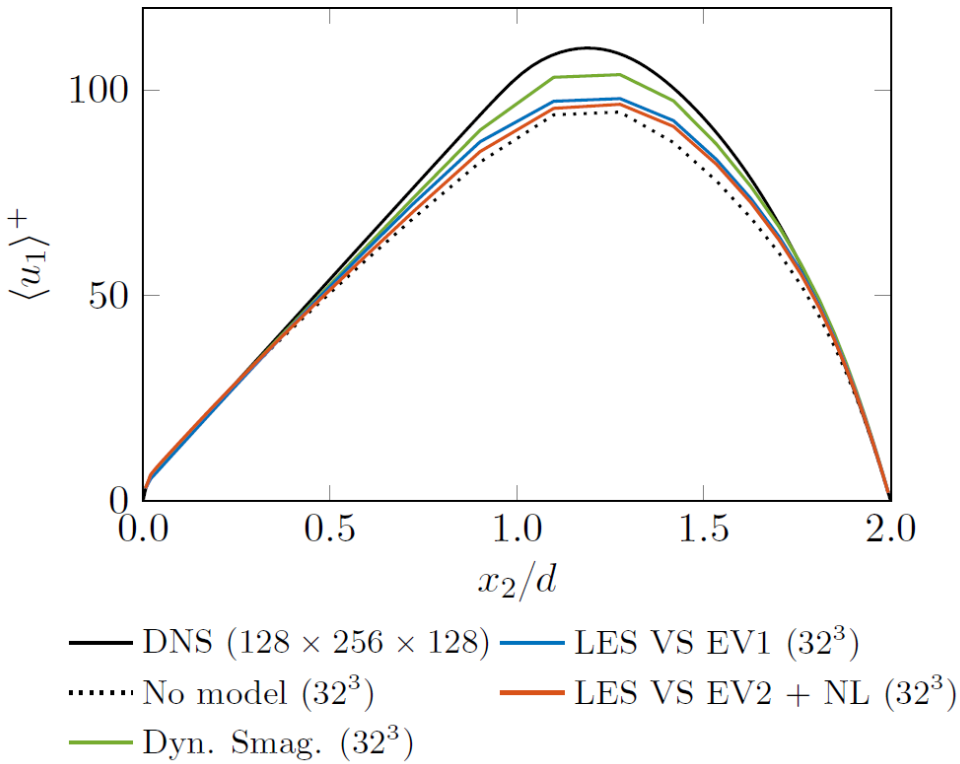
# Numerical results

## Large-eddy simulation with new nonlinear model

- Mean velocity




- Observations
  - Dynamic Smagorinsky model provides best prediction
  - Eddy viscosity model close to no-model result
  - Nonlinear model does not improve or deteriorate this result
  - Similar observations for $Ro^+ = 50, 75, 100, \ldots, 250$
  - Similar observations for $n_x \times n_y \times n_z = 64^3$

$Re_\tau \approx 395$, $Ro^+ = 100$

Silvis *et al.* (2017), *APS DFD Meeting*

# Numerical results

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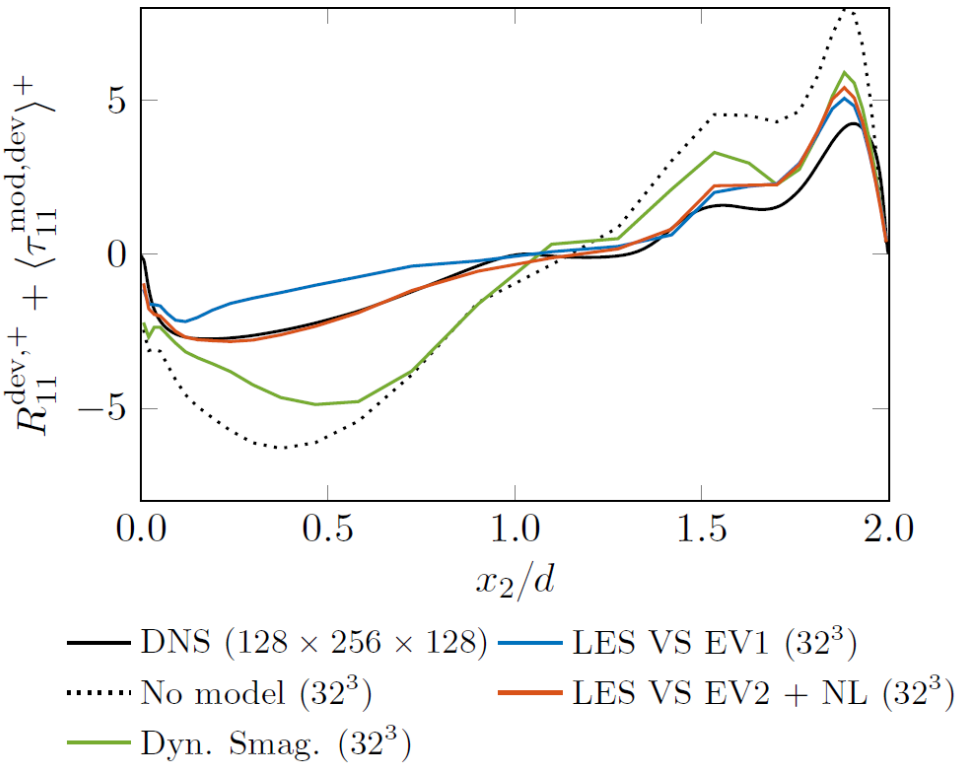
- Streamwise stress anisotropy




- Observations
  - Dynamic Smagorinsky prediction much less good
  - Nonlinear model improves shape and magnitude of stresses a lot
  - Similar observations for $Ro^+ = 50, 75, 100, \ldots, 250$, for the same model constants
  - Similar observations for $n_x \times n_y \times n_z = 64^3$, for the same model constants

Silvis *et al.* (2017), *APS DFD Meeting*

$Re_\tau \approx 395$, $Ro^+ = 100$

# Numerical results

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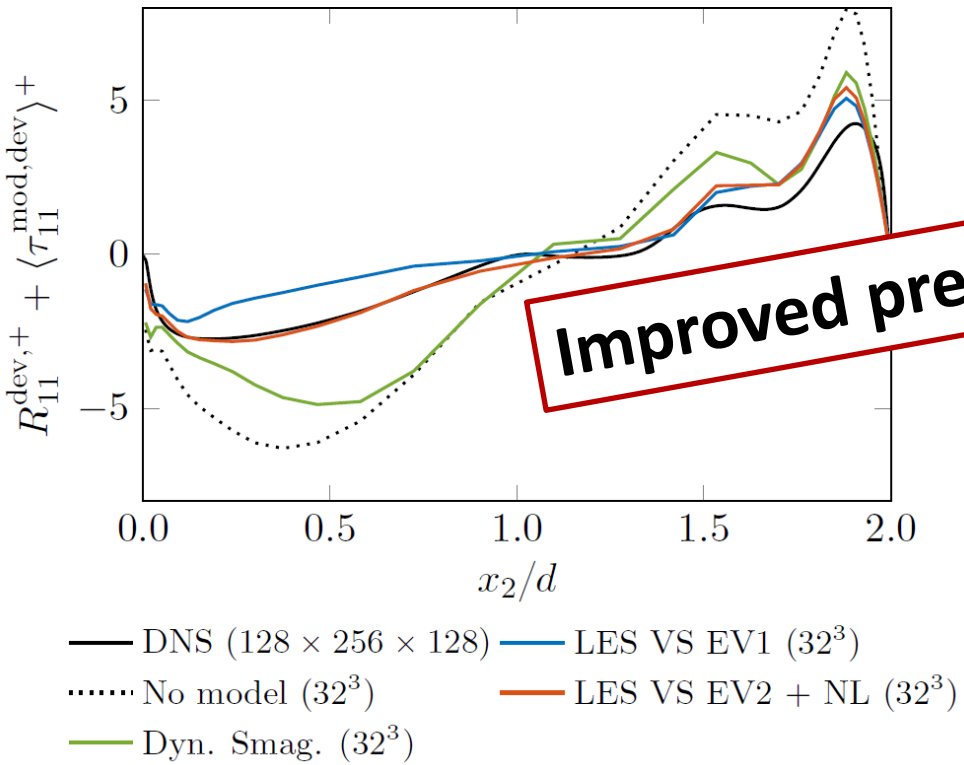


**Improved prediction**

- Observations
  - Dynamic Smagorinsky prediction much less good
  - Nonlinear model improves shape magnitude of stresses a lot
  - Similar observations for $Ro^+ = 50, 75, 100, \ldots, 250$, for the same model constants
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$Re_\tau \approx 395$, $Ro^+ = 100$

Silvis *et al.* (2017), *APS DFD Meeting*

# Summary and outlook

## Conclusions

- Spanwise-rotating turbulent plane-channel flows
  - A rich test case
  - Ample opportunity for large-eddy simulation

- New nonlinear subgrid-scale model
  - Improves predictions of the Reynolds stress anisotropy
  - No negative effects on the mean velocity

## Outlook

- Further study of the nonlinear subgrid-scale model
  - Combine dynamic Smagorinksy model with nonlinear term
  - Combine with different subgrid characteristic length scale, $\delta$

Silvis *et al.* (2016), *Proceedings of the CTR Summer Program*, 265

Trias *et al.* (2017), to appear in *Phys. Fluids* **29**

# Thank you for your attention!

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