Symmetries and conservation laws as constraints for the modeling and numerical simulation of turbulent flows

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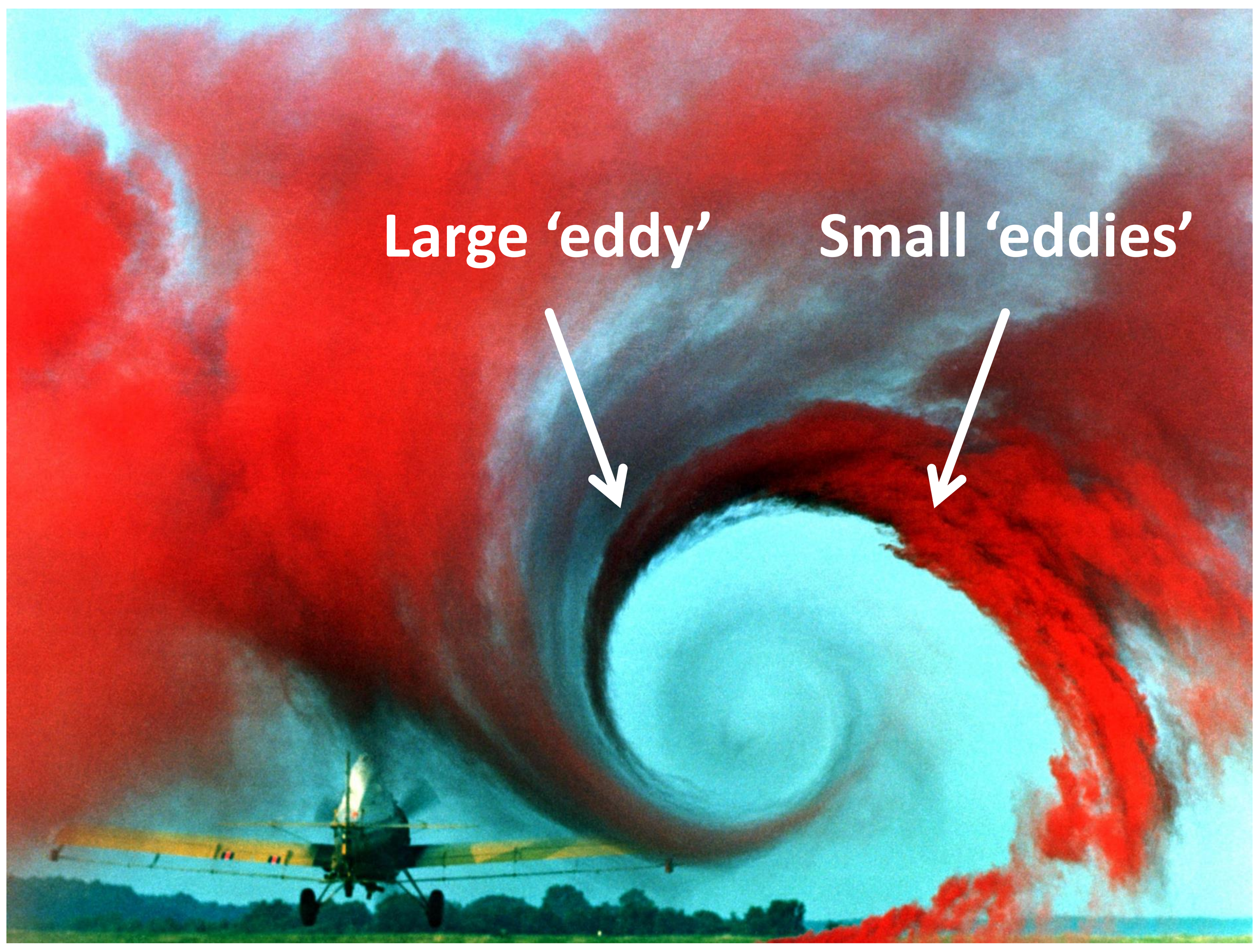
Abstract Numerical solution of the equations of fluid dynamics, the Navier–Stokes equations, is not feasible for most practical turbulent flows, due to high resolution requirements. Therefore, usually turbulence models are employed. We present a framework of constraints for the assessment and creation of turbulence models, based on the idea of preserving the symmetries and other fundamental properties of the Navier–Stokes equations and the turbulent stresses. We also present conservation constraints for discretizations of the Navier–Stokes equations. Finally, we wonder how the symmetries of the Navier–Stokes equations can be satisfied on the discrete level.

Turbulent flows

Goal

- Numerical prediction of turbulent flows



Challenges

- The small eddies cannot be resolved
  - Only compute the large eddies
- Closure problem
  - Model the small eddies

Research questions

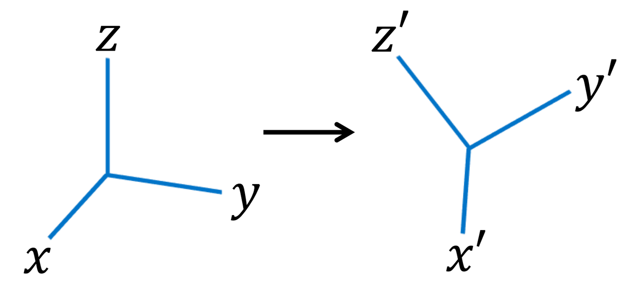
- How to model the small eddies?
- How to discretize the equations?

Preserve properties of the Navier–Stokes equations and the turbulent stresses

Model constraints

Properties of the Navier–Stokes equations

- Symmetries
  - Rotational invariance, Galilean invariance, …
- Conservation laws
  - Mass, momentum, angular momentum, vorticity-related quantities



Properties of the turbulent stresses

- Dissipation behavior
- Near-wall scaling behavior
- Realizability

Analysis of turbulence models

Model	Symmetries						Dissipation							
	Galilean	Rotation	Scaling	2DMFI	Time reversal	Conservation	2nd law	Vreman a	Vreman b	Nicoud *et al.* a	Nicoud *et al.* b	Verstappen	Near-wall scal.	Realizable
Smagorinsky	✓	✓	✗*	✓	✗*	✓	✓	✗	✓	✗	✗	✓	y^0	
WALE	✓	✓	✗*	✗	✗*	✓	✓	✗	✓	✗	✗	✓	y^3	
Vreman	✓	✓	✗*	✗	✗*	✓	✓	✗	✓	✗	✗	✓	y^1	
AMD	✓	✓	✗*	✓	✗*	✓	✓	✓	✗	✓	✗	✓	y^1	
Gradient	✓	✓	✗*	✗	✓	✓	✗	✓	✗	✓	✗			✓
Vortex stretch.	✓	✓	✗*	✓	✗*	✓	✓	✓	✗	✓	✓	✗	y^3	

✗* Dynamic procedure may restore symmetry

A framework for the assessment and creation of turbulence models

Discretization constraints

Properties of the Navier–Stokes equations

- Conservation laws
  - Mass, momentum, kinetic energy

Constraints on discrete operators

- Symmetries

How to preserve symmetries on the discrete level?

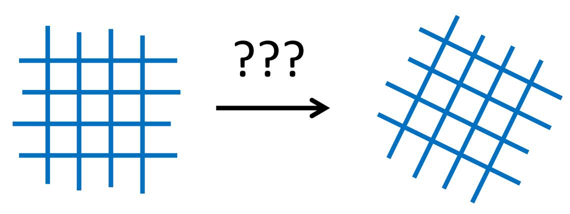


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Image: Wake Vortex Study at Wallops Island, NASA Langley Research Center

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