Symmetry-preserving subgrid-scale models for large-eddy simulation of turbulent flows

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Abstract In the current study we aim to go beyond the purely dissipative description of turbulent flows that is provided by eddy viscosity models for large-eddy simulation. We further aim to make this description physically consistent. Starting from a general subgrid-scale model that is nonlinear in the velocity gradient, we show how both dissipative and nondissipative processes can be represented. Then, several physical principles are outlined to restrict this general model to a class of symmetry-preserving subgrid-scale models, and existing models are compared with it. Finally, we wonder how these principles can be satisfied in numerical simulations.

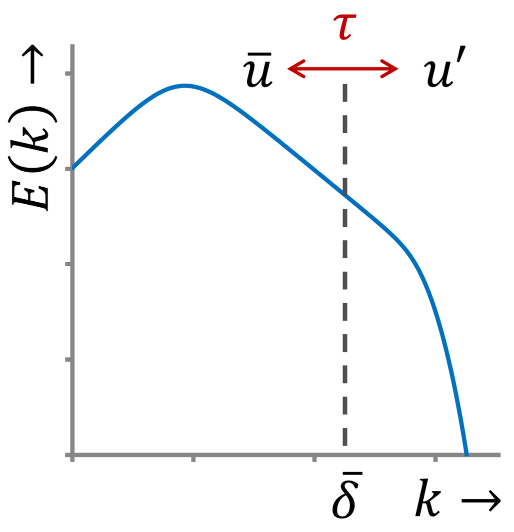
Introduction

Large-eddy simulation

- Calculate large scales

Subgrid-scale stresses τ?

- Model small scales



Research aims

- Describe dissipative and nondissipative processes

General nonlinear subgrid-scale models

- Physical consistency

Model constraints

General nonlinear subgrid-scale models

Assumption

$\tau^{mod} = f(\bar{S}, \bar{\Omega})$

Model building blocks

Rate of strain: $\bar{S}$

Rate of rotation: $\bar{\Omega}$

Possible model terms

$I,\ \bar{S},\ \bar{S}^2,\ \bar{S}\,\bar{\Omega},\ \bar{\Omega}\,\bar{S},\ \bar{\Omega}^2,\ \bar{S}^3,\ \bar{S}^2\,\bar{\Omega},\ \ldots$

Reduction of the number of model terms

- Cayley-Hamilton theorem
- Gram-Schmidt orthogonalization procedure

General nonlinear model

$$\tau^{mod} = \alpha_0 I + \underbrace{\alpha_1 \bar{S}}_{\text{Dissipative}} + \underbrace{\alpha_2 (\bar{S}^2)^\perp + \alpha_3 (\bar{\Omega}^2)^\perp + \cdots}_{\text{Nondissipative}}$$

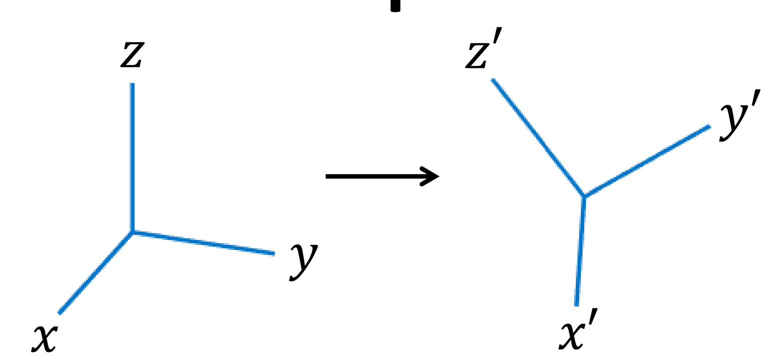
Model coefficients?

Model constraints

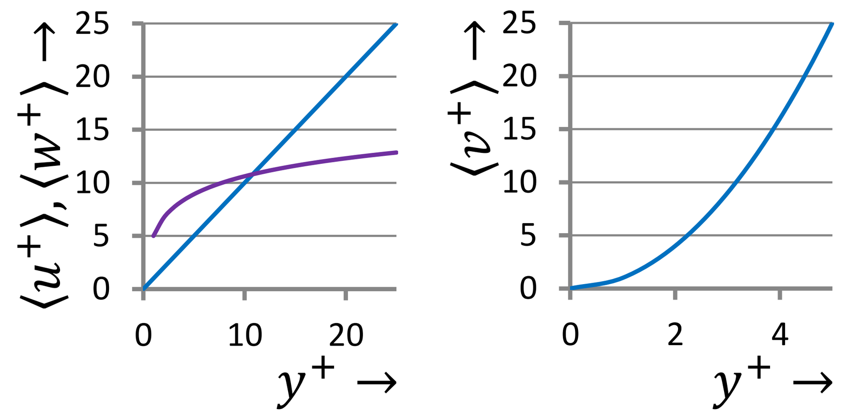
Model constraints

Symmetries of the Navier-Stokes equations

- Rotational invariance
- Galilean invariance
- Scaling invariance
- …



Near-wall scaling



Production of subgrid-scale kinetic energy

- Vreman's and Nicoud's laminar flow conditions
- Second law of thermodynamics

Class of symmetry-preserving subgrid-scale models

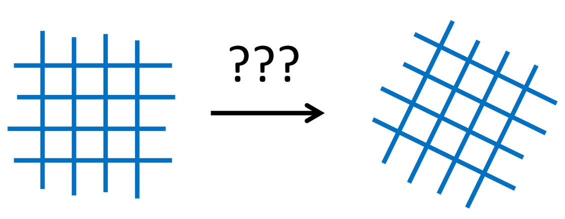
Analysis of existing models

Requirement	Smag.	WALE	Vreman	QR	S3PQR	Grad.
Rotational inv.	✓	✓	✓	✓	✓	✓
Galilean inv.	✓	✓	✓	✓	✓	✓
Scaling inv.	×*	×*	×*	×*	×*	×*
2D MFI	✓	×	×	✓	✓*	×
Near-wall scaling	$\mathcal{O}(y^0)$	$\mathcal{O}(y^3)$	$\mathcal{O}(y^1)$	$\mathcal{O}(y^1)$	$\mathcal{O}(y^3)$	×
Laminar flow $\nu_e = 0$	×	×	×	✓	✓*	✓
Turbulent flow $\nu_e \neq 0$	✓	✓	✓	×	×	×
Second law	✓	✓	✓	✓	✓	

×* Dynamic procedure may restore symmetry ✓* Depending on model parameter

Conclusions and future work

- We obtain a class of subgrid-scale models that
  - captures dissipative and nondissipative processes
  - is physically consistent
- More constraints for turbulence models?
- Measure of deviation from symmetry?
- Discrete level?



- *To appear soon: arXiv.org preprint*

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