A new subgrid characteristic length for large-eddy simulation

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Abstract

A new definition of the subgrid characteristic length, δ, for large-eddy simulation (LES) is proposed with the aim to answer the following research question: *can we find a simple and robust definition of δ that minimizes the effect of mesh anisotropies on the performance of subgrid scale (SGS) models?* In this regard, we consider the definition given by

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\mathsf{G}_\delta \mathsf{G}_\delta^T : \mathsf{G}\mathsf{G}^T}{\mathsf{G}\mathsf{G}^T : \mathsf{G}\mathsf{G}^T}}, \tag{1}$$

a very good candidate. Unlike the most common definitions in the context of LES that only depend on the mesh geometry, *i.e.* $\Delta \equiv \mathrm{diag}(\Delta x, \Delta y, \Delta z)$, it is also dependent on the local flow topology, $\mathsf{G} \equiv \nabla \overline{\boldsymbol{u}}$. The second-order tensor $\mathsf{G}_\delta \equiv \mathsf{G}\Delta$ can be viewed as the gradient in the so-called computational space. Actually, the definition of δ_{lsq} is obtained by minimizing (in a least-squares sense) the difference between the leading terms of the SGS tensor, $\tau(\overline{\boldsymbol{u}})$, for an isotropic filter length, $(\delta^2/12)\mathsf{G}\mathsf{G}^T + O(\delta^4)$ and anisotropic filter lengths, $(1/12)\mathsf{G}_\delta \mathsf{G}_\delta^T + O(\delta^4)$. This definition fulfills a set of desirable properties: locality, boundedness, low cost, sensitive to flow orientation... To get a better understanding of δ_{lsq}, results for a 2D simple flow are shown in Figure 1 (left). Interestingly, for $\omega = 1/2$ (simple shear flow) $\delta_{lsq} = \beta^{-1}$. This situation mimics the typical quasi-2D grid-aligned flow in the initial region of a shear layer. As could be expected, δ_{lsq} is equal to the grid size in the direction orthogonal to the shear layer. Finally, to show the adequacy of δ_{lsq} for highly anisotropic grids, LESs have been computed for (artificially) stretched meshes (see Figure 1, right). Notice that for increasing values of N_z, results with $\delta_{vol} = (\Delta x \Delta y \Delta z)^{1/3}$ diverge whereas results with δ_{lsq} rapidly converge. Results showing the performance of δ_{lsq} for more complex flows using advanced LES models [1] will be presented in the workshop.

Keywords: Large-eddy simulation; Subgrid characteristic length; Eddy-viscosity; Subgrid modeling; Turbulence;

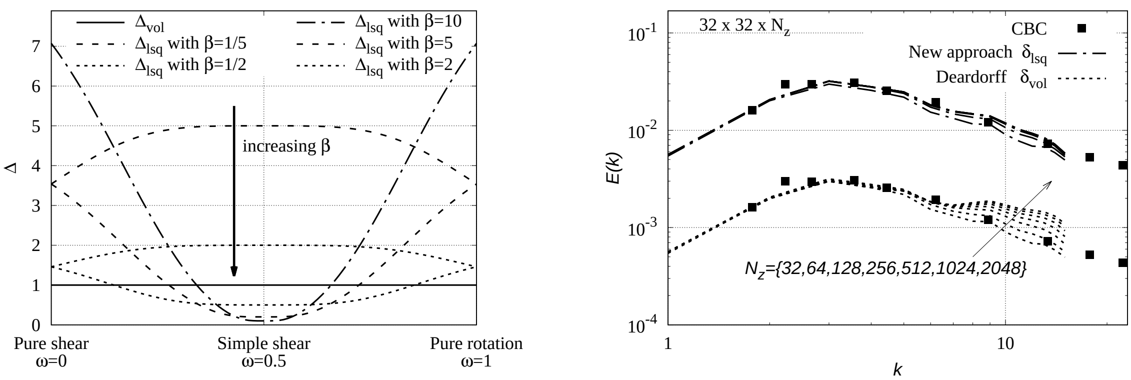

Figure 1: Comparison between the new definition δ_{lsq} proposed in Eq.(1) and the definition proposed by Deardorff [2], *i.e.* $\delta_{vol} \equiv (\Delta x \Delta y \Delta z)^{1/3}$. Left: a 2D simple flow given by $\Delta = \mathrm{diag}(\beta, \beta^{-1})$ and $[\mathsf{G}]_{1,1} = [\mathsf{G}]_{2,2} = 0$, $[\mathsf{G}]_{1,2} = 1$, $[\mathsf{G}]_{2,1} = 1 - 2\omega$. Right: energy spectra for decaying isotropic turbulence corresponding to the experiment of Comte-Bellot and Corrsin [3].

References

[1] Trias, F. X., Folch, D., Gorobets, A., and Oliva, A. *Physics of Fluids* **27**(6), 065103 (2015).

[2] Deardorff, J. W. *Journal of Fluid Mechanics* **41**, 453–480 (1970).

[3] Comte-Bellot, G. and Corrsin, S. *Journal of Fluid Mechanics* **48**, 273–337 (1971).

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