Osborne Reynolds

On the phenomenon of turbulence

BY **MAURITS SILVIS**

Osborne Reynolds can be seen as one of the founding fathers of modern turbulence research. During his almost-40-year-long academic career he worked on an wide range of physics and engineering problems, publishing more than 70 papers along the way. Two of these papers have had a very profound impact on the way we think about turbulence, and we'll look into these in the current article.

Osborne Reynolds was born in a well-established family in Belfast, Ireland, in 1842. Soon after, he and his family moved to the area of Essex, not far from Cambridge, where Reynolds grew up. His father, who was a reverend, was very much interested in mathematics and mechanics, and he took upon him the early education of his son. After being an apprentice in an engineering workshop, Reynolds decided to go to the University of Cambridge to study Mathematics. Not long after his graduation in 1867, Reynolds was appointed as a professor at Owens College in Manchester. He was only 25 at the time! He remained in Manchester until his retirement.

FIGURE 1 Osborne Reynolds (1842 -1912).

Turbulence and the Reynolds number

In his 1883 paper titled "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels" [2] Reynolds writes:

"The internal motion of water assumes one or other of two broadly distinguishable forms - either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths the most indirect possible."

In present-day terminology we call "direct" motion laminar, or layered. This is a state in which the behavior of a fluid is very regular. The sinuous paths Reynolds refers to are not sinusoidal (periodic) motions. Instead, they are the ones "having many curves and turns", being very irregular. We refer to these flows as turbulent, now.

Reynolds goes on to describe how one can distinguish these two flow types experimentally in a transparent medium like water.

"This [flow behavior] may be shown by adding a few streaks of highly coloured [sic] water to the clear moving water. Then although the coloured streaks may at first be irregular, they will, if there are no eddies [vortices], soon be drawn out into even colour bands; whereas if there are eddies they will be curled and whirled about in the manner so familiar with smoke."

With his experimental investigations he aimed to find out which quantities determine this distinction in flow behavior. Furthermore, he tried to explain what is causes a flow to transition from a laminar to a turbulent state.

As a first step, Reynolds notes that it is possible to rescale the Navier-Stokes equations, the equations describing the motion of fluids, in such a way that only a single dimensionless parameter, now called the Reynolds number, is involved. Denoting the rescaled (dimensionless) components of the velocity field by u_i and the rescaled pressure field by p , we may write

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j}.$$

According to the Einstein summation convention repeated indices imply a sum, here over the three directions of space. The Reynolds number, Re , depends on length ($\mathcal{L}$) and velocity ($\mathcal{U}$) scales that are typical for the flow at hand. The Reynolds number also depends on the kinematic viscosity (ν) of the fluid,

$$Re = \frac{\mathcal{U}\mathcal{L}}{\nu}.$$

Reynolds hypothesized that there is a critical flow velocity, u_c, that marks the transition between laminar (for flow velocities smaller than the critical velocity) and turbulent flows (for flow velocities larger than the critical velocity). Characterized by a critical Reynolds number, Re_c, it was thought to scale with the diameter of pipes used in experiments, d , and the fluid viscosity as

$$u_c = Re_c \frac{d}{\nu}.$$

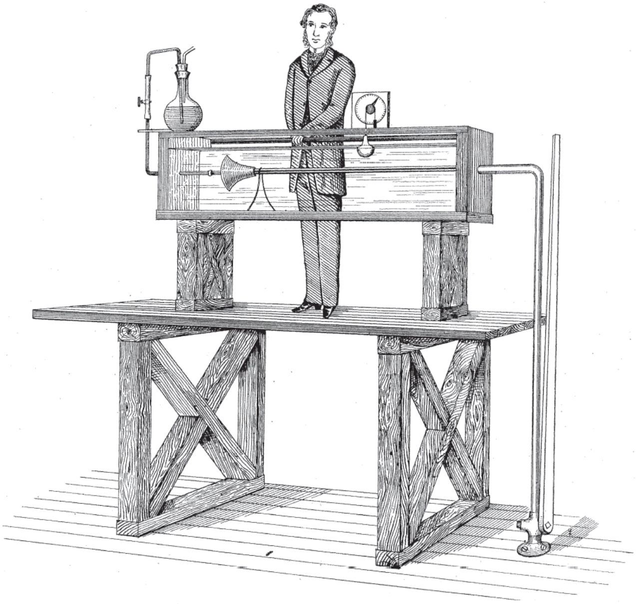

FIGURE 2 Schematical overview of the experimental setup that Reynolds used to verify his hypothesis.

The experimental setup that Reynolds used to verify his hypothesis is schematically shown in Fig. [[XX]]. It consisted of a large tank of water in which a glass tube could be placed. Water could flow from the tank into the tube via a large mouth piece, but only when the valve at the other end of the tube was opened. Using the valve, Reynolds set the rate at which water would flow through the tube. When the water was flowing, a dye would be 'pulled' inside the tube, working as a tracer of the fluid motion.

Reynolds separately varied the flow velocity, the tube diameter and the temperature of the water (effecting only the viscosity) to conclude that indeed there is a critical Reynolds number that determines if laminar or turbulent fluid motion was observed. Although Reynolds doesn't mention it in his paper, this critical Reynolds number was later estimated to be of the order of 2000.

Understanding turbulence

While establishing the role of the Reynolds number in his flow experiments, Reynolds remarks that his experimental results were very sensitive to initial disturbances in the flowing fluid:

"The critical velocity was very sensitive to disturbance in the water before entering the tubes; and it was only by the greatest care as to the uniformity of the temperature of the tank and the stillness of the water that consistent results were obtained. This showed that the steady motion was unstable for large disturbances long before the critical velocity was reached."

Also, he observed that, around the critical velocity, eddies would appear and grow rapidly, leading to a turbulent state in a very short time. He contrasts this with experiments in which he lets two streams flow in opposite directions in the same tube. In that case one can observe what is called a Kelvin-Helmholtz instability [3], during which vortices appear and grow gradually, seemingly independent of initial disturbances in both fluids.

Thus, Reynolds already noted that the mechanism by which vortices appear must be different in both experiments, and that the magnitude of disturbances plays a key role in causing turbulence, a remarkable observation given the fact that Reynolds lived over a 100 years ago. In present-day terminology the Kelvin-Helmholtz instability is called a linear instability. To indicate that the mechanism by which turbulence arises in regular pipe flow (a stream in one direction) is different, this latter mechanism is called a bypass transition. It is currently believed that this bypass transition can be 'delayed' almost indefinitely, by using smoother and smoother pipes and more regular inflow of fluid. Experiments confirm this feeling: laminar flows have been observed at Reynolds numbers as high as 10^7.

The attentive reader will notice that these ideas obfuscate the role of the Reynolds number as a precise indicator of the state of a flow. Indeed, it cannot be generally said that there is a single Reynolds number that marks the transition between laminar and turbulent flows, particularly if different flow geometries are considered. In practice you will therefore often find that the Reynolds number is only listed as a rough indication of flow behavior. Taking perturbations arising from initial and boundary conditions into account, we can say it has more power than that, however. If two flows have similar initial and boundary conditions (after making them dimensionless using typical length and velocity scales) and have a similar Reynolds number, then these flows can be expected to show the same behavior.

Predicting turbulence

In a follow-up paper titled "On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion" [4] Reynolds aims to theoretically explain, on the basis of the Navier-Stokes equations, when a fluid flow is laminar and when it can become turbulent.

Unfortunately, the paper is very hard to read and confusing in its terminology. Even Sir George Gabriel

Stokes, who was one of the eminent scientists reviewing the paper, found this, as can be deduced from a letter he wrote [1]:

"Lensfield Cottage, Cambridge, 31 Oct. 1894

Dear Lord Rayleigh,

I must plead guilty to not having digested Professor Osborne Reynolds's paper, though much time has passed since it was referred to me.

I find it very difficult to make out what the author's notions are. As far as I can conjecture his meaning, I must say I do not think he has made out his point. He is however an able man, and in his former paper did very good work […]. The fact that the author has gone to the expense of printing the paper shows that he himself considers it as of much importance. I confess I am not prepared to endorse that opinion myself, but neither can I say that it may not be true.

[…]

Yours very truly,

G. G. Stokes"

Despite this review, Reynolds' paper was published and it is now seen as a landmark contribution to the field of fluid mechanics, and particularly to the research relating to the behavior of turbulent flows. In the paper, Reynolds proposed to take the Navier-Stokes equations as a starting point to derive equations for the evolution of the mean, or average, velocity field. Furthermore, he obtained equations for the evolution of the kinetic energy contained in the mean velocity field, and for the energy contained in the small fluctuating motions. Reynolds argued on the basis of these equations that a flow would be laminar if the energy contained in the small-scale motions does not increase, that is, if at least as much energy is dissipated due to friction (viscosity) as is transported from the large to the small motions. For a flow between two parallel plates, he then shows that laminar flow can be expected for all Reynolds numbers less than 517.

This reasoning and the result are not currently seen as the most important result of that paper. The procedure in which the velocity field is decomposed into a mean and a fluctuating part is. It is now referred to as Reynolds averaging. Although in a slightly modified form, it constitutes the basis for modern simulation techniques like the Reynolds-averaged Navier-Stokes (RANS) approach and large-eddy simulation (LES). There, one of the chief problems is that the evolution of the mean velocity field, or of the larger scales of motion within a flow, is not independent of what happens on small scales. But that's a story I'll leave for some other time.

Conclusions

During his life, Osborne Reynolds made very important contributions to the understanding of turbulent flows. In the current article we looked at his experimental work, in which he discovered a single dimensionless quantity that characterizes flow behavior as laminar or turbulent - although disturbances caused by initial and boundary conditions play an important role also. In the theoretical paper we subsequently discussed, Reynolds aimed to find an explanation for his observations. Although that goal may not have been reached, he did provide the basis for simulation techniques used in modern turbulence research. Furthermore, in both papers, Reynolds addresses a few fundamental questions relating to the mechanisms behind and causes of turbulence, some of which have still not been fully answered today •

References

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[4] Reynolds, O. (1895), Phil. Trans. R. Soc. Lond. 186, 123-164, http://rsta.royalsocietypublishing.org/content/186/123