Going beyond eddy viscosity: Finding a minimal representation of subgrid-scale stresses in large-eddy simulation

Maurits H. Silvis*, Roel W.C.P. Verstappen

Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, The Netherlands

Introduction In the current study we aim to go beyond the dissipative description of turbulent flows provided by eddy viscosity models for large-eddy simulation. As a starting point, we consider a general subgrid-scale model that is nonlinear in the velocity gradient. To reduce the number of degrees of freedom of the model, we propose a first-principles-based procedure to find a minimal representation of subgrid-scale stresses. Also, several criteria to determine the dependence of model coefficients on flow properties are put forward. Ultimately, this would lead to a better understanding of the role of different nonlinear model terms in the description of turbulent flows.

Large-eddy simulation

Filtered Navier-Stokes equations

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\tau_{ij}$$

- Subgrid-scale stress tensor: $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$
- Closure problem: model $\tau^{mod} \approx \tau$

Model building blocks

Rate of strain: $\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$

Rate of rotation: $\bar{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i}\right)$

Examples of subgrid-scale models

Eddy viscosity models

$$\tau^{mod} - \tfrac{1}{3}\mathrm{tr}(\tau^{mod}) I = -2\nu_e \bar{S}$$

Energy transfer from large to small scales:

$$\mathcal{P}^{mod} = -\mathrm{tr}(\tau^{mod}\bar{S}) = 2\nu_e \mathrm{tr}(\bar{S}^2)$$

- + Mean dissipation captured for $\nu_e > 0$
- – Backward scatter only possible for $\nu_e < 0$
- – Stress structure imposed incorrectly

The gradient model

$$\tau^{mod} = C(\bar{S}^2 - \bar{\Omega}^2 - (\bar{S}\,\bar{\Omega} - \bar{\Omega}\,\bar{S}))$$

Energy transfer from large to small scales:

$$\mathcal{P}^{mod} = -\mathrm{tr}(\tau^{mod}\bar{S}) = -C(\mathrm{tr}(\bar{S}^3) - \mathrm{tr}(\bar{S}\,\bar{\Omega}^2))$$

- + Represents forward and backward scatter
- + Stress structure resembled better
- – Not enough forward scatter, unstable

Mixed models

Linear combination of the above models

- + Stable model that represents forward and backward scatter
- – Somewhat *ad hoc* combination of models

General nonlinear subgrid-scale model

Assumption

$$\tau^{mod} - \tfrac{1}{3}\mathrm{tr}(\tau^{mod}) I = f(\bar{S}, \bar{\Omega})$$

Cayley-Hamilton theorem

$$\tau^{mod} = c_1 \bar{S} + c_2 \bar{S}^2 + c_3 \bar{\Omega}^2 + c_4(\bar{S}\,\bar{\Omega} - \bar{\Omega}\,\bar{S}) + c_5(\bar{S}^2\bar{\Omega} - \bar{\Omega}\,\bar{S}^2) + \cdots$$

⟵ New terms

Improved representation of subgrid-scale stresses?

What do the terms represent?

$$\mathcal{P}^{mod} = -\mathrm{tr}(\tau^{mod}\bar{S}) = -c_1\mathrm{tr}(\bar{S}^2) - c_2\mathrm{tr}(\bar{S}^3) - c_3\mathrm{tr}(\bar{S}\,\bar{\Omega}^2) - c_4\mathrm{tr}(\bar{S}^2\bar{\Omega} - \bar{\Omega}\,\bar{S}^2) - c_5\mathrm{tr}(\bar{S}^3\bar{\Omega} - \bar{\Omega}\,\bar{S}^3) - \cdots$$

Energy transfer from large to small scales (the c_1, c_2, c_3 terms)

Distribution of energy among large scales? (the c_4, c_5 terms)

Minimal representation of subgrid stresses

- Find independent terms from Gram-Schmidt process
- Select smallest to represent τ

Model too complex?

How to set the coefficients?

- Analytical requirements:

  $\tau_{ij}\bar{S}_{ij} = 0 \Rightarrow \tau^{mod}_{ij}\bar{S}_{ij} = 0$

  $\frac{\partial}{\partial x_j}\tau_{ij} = 0 \Rightarrow \frac{\partial}{\partial x_j}\tau^{mod}_{ij} = 0$

  Which flows satisfy this?

  Preservation of symmetries
- Estimate from data: *a priori* analysis

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* Contact: m.h.silvis@rug.nl

university of groningen / faculty of mathematics and natural sciences
NWO Netherlands Organisation for Scientific Research
J.M.Burgerscentrum Research School for Fluid Mechanics