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Symmetry constraints for the modeling and numerical simulation of turbulent flows

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Abstract It is well known that the governing equations of fluid dynamics, the Navier–Stokes equations, are invariant under certain transformations, such as instantaneous rotations of the coordinate system and the Galilean transformation. These transformations, also referred to as symmetries of the equations, play an important physical role because they ensure that the description of fluids is the same in all inertial frames of reference. Furthermore, they relate to conservation and scaling laws. It has since long been realized that it is desirable that these symmetries are also satisfied in large-eddy simulations [1, 4].

Using large-eddy simulations one aims to predict the large-scale behavior of turbulent flows. This is done through numerical solution of the Navier–Stokes equations, on grids that are too coarse to resolve all the relevant physical details. An extra forcing term called a turbulence, or subgrid-scale, model is introduced to represent the effects of the (unresolved) small scales on the (resolved) large-scale motions.

We present a framework of constraints for the creation and assessment of subgrid-scale models for large-eddy simulation [2, 3], based on the idea that it is desirable that subgrid-scale models are consistent with the symmetries, as well as with other mathematical and physical properties of the Navier–Stokes equations. We further discuss issues of numerical implementation, including that of conservation of energy in simulations [5], and we show results of large-eddy simulations of turbulence, obtained using a new subgrid-scale model with built-in desirable properties.

References

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