

# Nonlinear Subgrid-Scale Models for Large-Eddy Simulation of Rotating Turbulent Flows

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**Abstract** We aim to design subgrid-scale models for large-eddy simulation of rotating turbulent flows. Rotating turbulent flows form a challenging test case for eddy viscosity models due to the presence of the conservative Coriolis force. We therefore propose a new subgrid-scale model that, in addition to a dissipative eddy viscosity term, contains a nondissipative nonlinear model term that can capture transport processes, such as those due to rotation. We show that the addition of this nonlinear model term leads to improved predictions of the Reynolds stress anisotropy in large-eddy simulations of a spanwise-rotating plane-channel flow, while maintaining the prediction of the mean velocity profile that is obtained when only using an eddy viscosity model.

## 1 Introduction

We consider large-eddy simulation of incompressible rotating turbulent flows. In large-eddy simulation one seeks to predict the large-scale behavior of turbulent flows without resolving all the relevant flow details. This is commonly done by supplementing the Navier–Stokes equations with an additional forcing term, a subgrid-scale model, aimed at representing the unresolved flow physics.

Rotating turbulent flows form a challenging test case for large-eddy simulation due to the presence of the Coriolis force. The Coriolis force conserves the total kinetic energy, while also redistributing it. More specifically, the Coriolis force transports kinetic energy from small to large scales of motion, leading to the formation of large-scale anisotropic structures [7]. Many subgrid-scale models for large-eddy simulation are, however, (primarily) designed to parametrize the dissipative nature of turbulent flows, ignoring transport processes.

We therefore consider a subgrid-scale model consisting of two terms. The first term is of eddy viscosity type. This term is linear in the rate-of-strain tensor and is used to represent the dissipative behavior of turbulent flows. The second term is nonlinear in the local velocity gradient and is aimed at parametrizing nondissipative processes, such as those due to rotation. We study the behavior of this nonlinear subgrid-scale model in large-eddy simulations of a spanwise-rotating plane-channel flow.

The structure of this paper is as follows. The nonlinear subgrid-scale model for large-eddy simulation is introduced in Section 2. Then, Section 3 describes the details of spanwise-rotating plane-channel flows, of which we perform large-eddy simulations in Section 4. Finally, conclusions are drawn in Section 5.

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## 2 Nonlinear Subgrid-Scale Models

Large-eddy simulations of incompressible rotating turbulent flows can be described by

$$\frac{\partial u_i}{\partial x_i} = 0, \quad \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + 2\nu \frac{\partial}{\partial x_j} S_{ij} - 2\epsilon_{ijk} \Omega_j u_k - \frac{\partial}{\partial x_j} \tau_{ij}^{\text{mod}}. \quad (1)$$

Here,  $u_i$  indicates the  $x_i$ -component of the large-scale velocity field, while  $p$  represents the modified large-scale pressure, including the centrifugal force. The density and kinematic viscosity are labeled  $\rho$  and  $\nu$ , respectively. The rate-of-strain and rate-of-rotation tensors of the large-scale velocity field are defined according to

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right), \quad (2)$$

while  $\Omega_i$  represents the rotation rate of the frame of reference about the  $x_i$ -axis. Without loss of generality we will assume that the axis of rotation is the  $x_3$ -axis, i.e.,  $\Omega_i = \delta_{i3} \Omega_3$ . The Einstein summation convention is assumed for repeated indices. Note that we consider large-eddy simulation without explicit filtering. Hence, no bars or tildes indicating a filtering operation appear in the above equations.

We model the deviatoric part of the subgrid-scale stress tensor with the following nonlinear model,

$$\tau^{\text{mod,dev}} = -2\nu_e S + \mu_e (SW - WS). \quad (3)$$

The first term on the right-hand side of Eq. (3), the usual eddy viscosity term, is used to parametrize dissipative processes in turbulent flows. The second term, that is nonlinear in the velocity gradient, is added because it is perpendicular to the rate-of-strain tensor. Therefore, it does not directly contribute to the subgrid dissipation and it represents energy transport. As this term contains the rate-of-rotation tensor, it has ‘‘a particular potential for [the simulation of] rotating flows’’ [4].

We propose to define the eddy viscosity,  $\nu_e$ , and the transport coefficient,  $\mu_e$ , by

$$\nu_e = (C_\nu \delta)^2 \frac{1}{2} |S| f_{\text{VS}}^3, \quad (4)$$

$$\mu_e = C_\mu \delta^2 \frac{1}{4} f_{\text{VS}}^4. \quad (5)$$

Here,  $C_\nu$  and  $C_\mu$  are the model constants,  $\delta$  represents the subgrid characteristic length scale and the magnitude of the rate of strain is defined as  $|S| = \sqrt{\text{tr}(S^2)}$ . The nondimensionalized vortex stretching magnitude,

$$f_{\text{VS}} = \frac{|S\vec{\omega}|}{|S||\vec{\omega}|}, \quad (6)$$

is used to enforce the proper near-wall scaling behavior of the modeled stresses and to make sure that the model vanishes in two-component flows [5]. The vorticity vector is given by  $\omega_i = -\epsilon_{ijk} W_{jk}$ .

## 3 Spanwise-Rotating Plane-Channel Flow

To study the vortex-stretching-based nonlinear subgrid-scale model of Eq. (3) we consider large-eddy simulations of a spanwise-rotating plane-channel flow. Such a flow can be characterized using the friction Reynolds and rotation numbers,

$$Re_\tau = \frac{u_\tau d}{\nu}, \quad Ro_\tau = \frac{2\Omega_3 d}{u_\tau}, \quad (7)$$

where  $u_\tau$  is the friction velocity and  $d$  represents the channel half-width.

The rotation number,  $Ro_\tau$ , determines the behavior of a spanwise-rotating plane-channel flow. For small rotation numbers the flow is mostly turbulent, although a (small) laminar region may appear close to one of the walls. As the rotation number increases, the laminar flow portion grows, until, for significant rotation numbers, the flow fully laminarizes. The mean velocity profile of a spanwise-rotating plane-channel flow exhibits a characteristic linear slope (proportional to  $Ro_\tau$ ) corresponding to the unstable (turbulent) part of the flow, while a parabolic profile emerges on the stable (laminar) side. Laminarization is further characterized by the decay of the Reynolds stresses. Refer to the work by Grundestam et al. [3] for more information about spanwise-rotating plane-channel flows.

## 4 Numerical Results

We studied the vortex-stretching-based nonlinear subgrid-scale model of Eq. (3) by performing direct and large-eddy simulations of a spanwise-rotating plane-channel flow with  $Re_\tau \approx 395$  and a moderate rotation number,  $Ro_\tau = 100$ . These simulations were performed using an incompressible Navier–Stokes solver employing a kinetic-energy-conserving spatial discretization of finite-volume type [9]. As such, the kinetic energy in the simulations was by construction conserved by convection, by the Coriolis force and by the nonlinear term of the subgrid-scale model.

The flow domain in the simulations had dimensions  $2\pi d \times 2d \times \pi d$  and was taken periodic in the streamwise ( $x_1$ ) and spanwise ( $x_3$ ) directions. The large-eddy and direct numerical simulations were, respectively, performed on  $32^3$  and  $128 \times 256 \times 128$  grids that were stretched in the wall-normal direction.

The large-eddy simulations made use of the vortex-stretching-based eddy viscosity model (Eq. (3) with  $C_\mu = 0$ ) and the vortex-stretching-based nonlinear subgrid-scale model of (Eq. (3) with  $C_\mu \neq 0$ ). The value of the eddy viscosity constant was estimated to be  $C_\nu \approx 0.59$  by requiring that the average dissipation due to the model matches the average dissipation of the Smagorinsky model [5]. The value of the transport coefficient of the nonlinear model,  $C_\mu$ , was subsequently tuned to obtain the best prediction of the Reynolds stresses. As is commonly done, the subgrid characteristic length scale was defined using the local grid size,  $\delta = (\Delta x_1 \Delta x_2 \Delta x_3)^{1/3}$  [2]. Refer to the literature for an overview of alternative (flow-dependent) definitions of this length scale [6, 8]. Results from direct numerical simulations and from large-eddy simulations without a subgrid-scale model serve as reference data.

Figure 1 shows the mean velocity profile, and the behavior of the Reynolds shear stress and spanwise Reynolds stress as obtained from the simulations. Since we consider traceless subgrid-scale models, only the deviatoric (anisotropic) part of the Reynolds stresses is considered. These results are further compensated by the average contribution from the subgrid-scale model [10].

The typical features of the flow in a spanwise-rotating plane-channel are clearly visible: the mean velocity profile exhibits a linear slope on the unstable side of the channel, while the Reynolds shear stress attains small values on the stable side. Contrary to what could be expected, the stresses do not exactly vanish on the stable side of the channel, which is most likely due to the occurrence of turbulent bursts [1].

The large-eddy simulations with the vortex-stretching-based subgrid-scale models slightly improve the prediction of the peak height and slope of the mean velocity profile when compared to the no-model result. Corresponding behavior can be observed in the Reynolds shear stress. These results indicate that the vortex-stretching-based eddy viscosity and nonlinear subgrid-scale models behave well.

The added value of these subgrid-scale models becomes clear when considering the deviatoric part of the streamwise Reynolds stress. Large-eddy simulations without a subgrid-scale model fail to predict that quantity, supporting the conclusion that subgrid-scale modeling is indeed justified, even at low friction Reynolds numbers [6]. Large-eddy simulations with the vortex-stretching-based eddy viscosity model provide a reasonable prediction of the streamwise Reynolds stress. This prediction is improved when including the nonlinear model term, as can most clearly be seen on the unstable

side of the channel ( $0 \leq x_2/d \leq 1$ ). Similar conclusions can be drawn for the deviatoric part of the wall-normal and spanwise Reynolds stresses (not shown). Thus, the addition of the nonlinear term to an eddy viscosity model leads to an improved prediction of the Reynolds stress anisotropy, while maintaining a reasonable prediction of the mean velocity profile and the Reynolds shear stress.

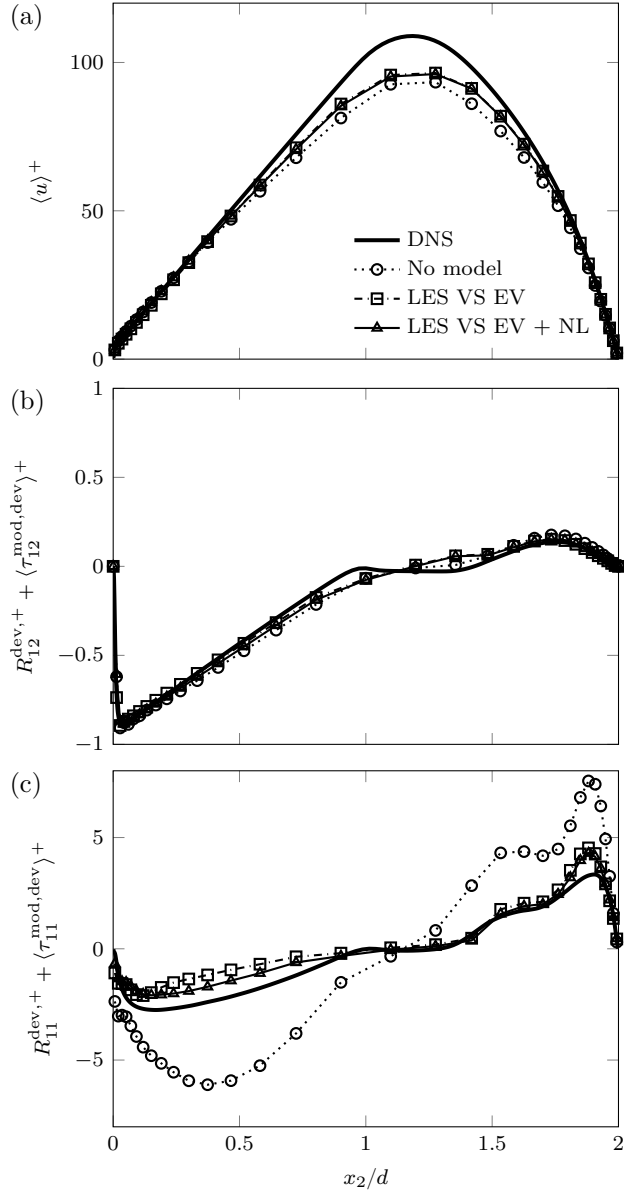


Figure 1: **a** Mean velocity profile, **b** Reynolds shear stress compensated by the model contribution and **c** deviatoric part of the streamwise Reynolds stress compensated by the model contribution, as obtained from large-eddy simulations (LES) of a spanwise-rotating plane-channel flow at  $Re_\tau \approx 395$  and  $Ro_\tau = 100$  on a  $32^3$  grid. Simulations were performed without a subgrid-scale model (*dotted line, circles*), with the vortex-stretching-based eddy viscosity (VS EV) model (Eq. (3) with  $C_\nu \approx 0.59$  and  $C_\mu = 0$ ) (*dashed line, squares*), and with the vortex-stretching-based nonlinear (VS EV + NL) subgrid-scale model (Eq. (3) with  $C_\nu \approx 0.59$  and  $C_\mu = 5$ ) (*solid line, triangles*). Results from direct numerical simulations (DNS) on a  $128 \times 256 \times 128$  grid are shown as reference (*thick solid line*). The quantities on the vertical axis are nondimensionalized using the friction velocity.

## 5 Conclusions

We focused on the construction of subgrid-scale models for large-eddy simulation of rotating turbulent flows. Rotating turbulent flows are characterized by the presence of the conservative Coriolis force. These flows form a challenging test case for large-eddy simulations using eddy viscosity models, as these subgrid-scale models are mainly aimed at capturing the dissipative behavior of turbulent flows. We therefore proposed a new subgrid-scale model that, in addition to a dissipative eddy viscosity term, contains a nondissipative nonlinear term. This subgrid-scale model was successfully tested in large-eddy simulations of a spanwise-rotating plane-channel flow. In particular, we showed how the addition of the nonlinear model term leads to an improved prediction of the Reynolds stress anisotropy. These findings confirm the potential of a nondissipative nonlinear model term for large-eddy simulation of rotating turbulent flows.

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